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QUALITATIVE ANALYSIS ON EXISTENCE OF FIRST ORDER NEUTRAL DELAY DIFFERENCE EQUATIONS

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ABSTRACT

In this paper, some oscillatory criteria for the solutions of first order neutral delay difference equation with negative coefficient are obtained. Here, I consider the neutral delay difference equation of the form,

$$\Delta(x_n+r_nx_{n-k})-q_ng(x_{n-m}) = 0, \quad (1.1)$$

Where, $n\varepsilon N(n_0)$, n_0 is a nonnegative integer and Δ is the forward difference operator.$\{q_n\}$ is the positive sequences and $\{r_n\}$ is a real sequence, g is the continuous functions such that $ug(u)\neq 0$, for $u\neq 0$. Also k, m are positive integers. Here, Δ is the forward difference operator defined by, $\Delta y_n= y_{n+1}-y_n$. By a solution of (1.1), we mean a real sequence $\{x_n\}$ which satisfies the equation(1.1) for all $n\varepsilon N_0$. A solution $\{x_n\}$ of (1.1) is said to be oscillatory, if the terms of the sequence are not eventually positive, or not eventually negative. Otherwise, it is called non-oscillatory. Equation (1.1) is said to be oscillatory, if all its solutions are oscillatory. Following this trend, in this paper, some sufficient conditions for oscillation of all solutions of equation (1.1) is obtained.

KEYWORDS: Oscillation, Neutral & Difference Equations

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1. INTRODUCTION

In recent years, the literature on the oscillation theory of neutral delay difference equations is growing very fast. This is due to the fact that the neutral delay difference equations are a new field with interesting applications in real world life problems. The obtained criteria improve some known results in the oscillatory behavior of solutions of first order neutral delay difference equations. See for example [1-11] and the references cited therein.

2. MAIN RESULTS

In this section, some sufficient conditions for the oscillation of all solutions of equation (1.1) are obtained. Let us assume the following conditions.

- **A_1**: $H_{n-a+l-m}= q_{n-2a-m}-q_n-q_{n-a-m}$ and $H_{n-a+l-m}\geq 0$.
- **A_2**: $r_{n-a-m}H_{n-a+l-m}\leq h_1$, $q_{n-m}[H_{n-a+l-m}]\leq h_2$, , for h_1 , $h_2\varepsilon N(n_o)$

Lemma2. 1[4]

Suppose, there exists a real number $a\varepsilon N(n_o)$, such that

$z_a(n) = r_n - \sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m} \geq 1.$

Let x_n be an eventually positive solution of the difference inequality,

$\Delta\,(x_{+} r_n x_{n-k}) - q_n x_{n-m} \geq 0$ (2.1)

Where,

$$z_n = x_n + r_n x_{n-} + \sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m} \tag{2.2}$$

Then eventually $\Delta z_n \geq 0$ and $z_n \leq 0$.

Proof

To prove $\Delta z_n \geq 0$

$\Delta z_n = \quad \Delta(x_{n+} r_n x_n) + \Delta\left(\sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m}\right)$

$= q_n x_{n-m} + q_{n-a-m} x_{n-2a-2m} + q_{n-2a-m} x_{n-2a-2m}$

$\Delta z_n \geq -H_{n-a+l-m} x_{n-a-m}$ (2.3)

$\Delta z_n \geq 0.$ (2.4)

To prove $z_n \leq 0$

Suppose $z_n > 0$, there exists $\mu > 0$ such that $z_n > \mu$

From(2.2), $x_n > \mu + r_n x_{n-} - \sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m}$

Let us consider two possible cases.

i)$\{x_n\}$ is unbounded, then $\lim_{n\to\infty} X_n = \infty$. Hence there exists a real sequence $\{v_i\}$, i=1,2, ….∞, such that $x(v_i) \to \infty$ as $i \to \infty$.

Let $x(v_i) = \max(x_n)$

$x(v_i) \geq \mu + r_n x_{n-} - \sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m}$

$x(v_i) \geq \mu + x(v_i)$, which is a contradiction. Hence $z_n \leq 0$.

ii)Suppose $\{x_n\}$is bounded., then $\lim_{n\to\infty} \sup x_n = L$.

Let $\{v_i\}$, i=1,2,…∞ , be a real sequence such that $x(v_i) \to L$, as $i \to \infty$.

Let $x(v_i) = \max(x_n)$

$x(v_i)\geq \mu + r_n x_{n-} - \sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m}$

$x(v_i)\geq \mu + x(v_i)$

$\sup x(v_i) \geq \sup(\mu + x(v_i))$, then $L \geq \mu + L$, which is a contradiction. Hence, $z_n \leq 0$.

Lemma2. 2[4]

Suppose, there exists a real number bεN(n_0) such that,

$Z_b(n)= r_n - \sum_{s=n-2b-m}^{n-m-b} q_s x_{s-m} \leq 1$.

Let x_n be an eventually negative solution of equation(2.1), and

Let $z_n=x_n-r_n x_{n-}+\sum_{s=n-2b-m}^{n-m-b} q_s x_{s-m}$

If the second order difference inequality $\Delta^2 x_n-(1/\rho_k)H_{n-b+l-m}\geq 0$,doesnot have

Proof

Suppose $z_n<0$, there exists $T<\rho_k$, such that $x_n<0$.

Let $M_1=\max x_n$

Let $z_n=x_n+r_n x_{n-}+\sum_{s=n-2b-m}^{n-m-b} q_s x_{s-m}$.

$x_T\leq M_1(r_{n-}-\sum_{s=n-2b-m}^{n-m-b} q_s x_{s-m})$

$x_T\leq M_1$, By induction we can prove $x_n \leq M_1$, for $T+n\rho_k\leq n\leq T+(n+1)\rho_k$.

Let $\lim_{n\to\infty} z_n=\alpha$,

i)let α=0.Then there exists $T_1<T$ such that $z_n\geq M_1$.,

$x_n\leq \{1/\rho_k\}\sum_{T_1}^{n+\rho_k} z_n$

ii)let α>0, Since $\Delta z_n\geq 0$, we have $z_n\leq\alpha$.

$x_n\leq \alpha+ r_n x_{n-}(\sum_{s=n-2b-m}^{n-m-b} q_s x_{s-m})$

$x_n \leq \alpha+ M_1(r_{n-}-\sum_{s=n-2b-m}^{n-m-b} q_s x_{s-m})$

$x_n \leq \alpha + M_1$

In general , $x_n \leq n\alpha + M_1$

Hence we have $\lim_{n\to\infty} x_n = \infty$. There exists $T_2 < T_1$, such that

$x_n \leq \{1/\rho_k\} \sum_{T_2}^{n+\rho_k} z_n$,combining both cases there exists $T^* < T_2$, such that $x_n \leq \{1/\rho_{k)}$

let $y_n = \sum_{T^*}^{n+\rho_k} z_n$, since $z_n < 0$, $y_n < 0$.

Hence we have, $\Delta y_n = z_{n,}$ $\Delta^2 y_n =$, $\Delta z_{n.}$

From(2.3), $\Delta z_n \geq -H_{n-b+l-m} x_{n-b-m}$

$$\Delta z_n \geq H_{n-b+l-m} \sum_{T^*}^{n+\rho_k} z_n$$

Hence, $\Delta^2 y_n - H_{n-b+l-m} \sum_{T^*}^{n+\rho_k} z_n \geq 0$, which is a contradiction to the given condition of lemma 2. 2. Hence, $z_n \geq 0$

Theorem2.1

Assume that { $q_{n-a-m}/(H_{n-a+l-2m})$} is a decreasing sequence and $A_{1,}$ A_2 holds. Then every solution of equation (1.1) is oscillatory.

Proof

Suppose to the contrary that equation (1.1) has eventually negative solution. Then, from lemma (2.2)

$$\Delta z_n \geq 0 \text{ and } z_n \geq 0. \tag{2.5}$$

Now, $\Delta z_n = \Delta(x_n + r_n x_n) + \sum_{s=n-2a-m}^{n-m-a} q_s x_{s-m}$

$\Delta z_n = q_n x_{n-m} + q_{n-a-m} x_{n-2a-2m} + q_{n-2a-m} x_{n-2a-2m}$

$\Delta z_n \geq -H_{n-a+l-m} x_{n-a-m}$

$\Delta z_n \geq -H_{n-a+l-m}[z_{n-a-m} + r_{n-a-m} x_{n-a-m-k} + q_s x_{s-m}]$

$\Delta z_n \geq H_{n-a+l-m} z_{n-a-m} - h_1 H_{n-a+l-m} x_{n-m-a-k} - H_{n-a+l-m}$

$\sum_{s=n-2a-m}^{n-m-a} q_s$ $(H_{s+l-m}/H_{s+l-m})\, x_{s-m}\}$ $_{-}H_{n-a+l-m}(H_{t+l-m}/H_{t+l-m})\, x_{t-l}\}$

$\Delta z_n \geq -H_{n-a+l-m} z_{n-a-m} + h_1 \Delta z_{n-k} + H_{n-a+l-m}(q_{n-a-m}/H_{n-a+l-2m})(z_n - z_{n-a-m})$

$\Delta z_n \geq -H_{n-a+l-m} z_{n-a-m} + h_1 \Delta z_{n-k} + h_2\, (z_{n-m} - z_{n-a-m}) + h_2(z_{n-a-m} - z_{n-l})$

$\Delta z_n \geq -H_{n-a+l-m} z_{n-l} + h_1 \Delta z_{n-k} + h_2 (z_{n-m} - z_{n-a-m}) + h_2 (z_{n-a-m} - z_{n-l})$

$(\Delta z_n - h_1 z_{n-k}) + (H_{n-a+l-m} + h_2) z_{n-l} - h_2 z_{n-m} \geq 0.$

Hence, by lemma 2.1, z_n is eventually negative solution. This is a contradiction to the equation (2.5). Hence, every solution of equation (1.1) oscillates.

3. CONCLUSIONS

In this paper, by using summation averaging techniques and comparison principle, some new oscillation criteria for first order neutral delay difference equation is obtained. This result improves some of the result obtained in [4].

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